Math in the Cosmos

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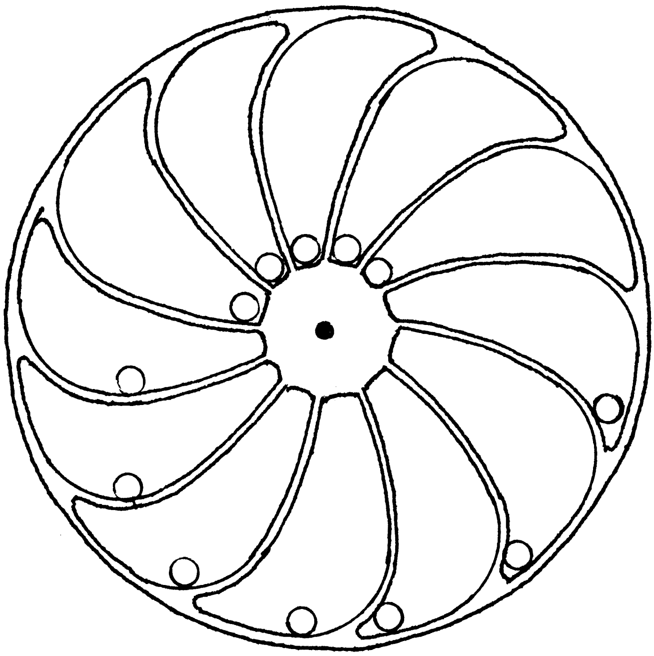
The Energy Drain

Impossible Machines

> IT IS NOT POSSIBLE TO BUILD A MACHINE THAT RUNS FOREVER WITH NO SOURCE OF ENERGY, YET PRODUCES USABLE ENERGY.

LIKE SQUARING THE CIRCLE, the problem of making a machine that would run forever has probably absorbed more man-hours than the building of the Egyptian pyramids. The idea of the "perpetuum mobile" is nearly as old as the pyramids. It is so seductive that I run a certain risk even writing about the subject. I mean, why shouldn't it be possible to build a machine that runs forever? I'll even throw in the requirement that it produces a little usable energy. In my mind's eye I see a simple but wonderful contrivance that will (in my mind) run forever. It is nothing more than a wheel with beautifully curved spokes, each engraved with a track that carries a steel ball.



A perpetual wheel

Clearly, the balls on the right-hand side of the wheel shown here are going to weigh more heavily, in turning the wheel, than those on the left. Hence, with barely a nudge, the wheel will begin to rotate. As each new spoke comes into position, another steel ball rolls to the right. In fact, because of the continuing downward acceleration of the right half of the wheel, it will spin with increasing velocity. One might imagine that it will spin faster and faster, without limit, until it literally flies into pieces. But at some point during this potentially fatal acceleration, the centrifugal force on the balls, particularly those on the left side, will prevent them from rolling toward the hub, as they would normally do. At such a pass, the wheel will continue to spin, gradually slowing, owing to frictional forces. As soon as the wheel slows enough to allow the balls on the left half of the wheel to roll to the right once more, it will, of course, begin to speed up again.

The wheel will therefore seek and find, ultimately, the narrowest range of speeds wherein it will remain spinning like the governor of a steam engine. Unlike a governor, however, this wheel has no external source of power! It will spin at exactly this rate until the parts begin to wear out. Sooner or later, something will give way and the wheel will roll to a clattering halt.

Nevertheless, it has been a valid demonstration of perpetual motion, at least conceptually. We could construct the wheel and its parts of modern space-age alloys, with Teflon bearings and silicon lubricants. It might be guaranteed to run for a thousand years. Or a million. Put enough care into it, and the latest version of the wheel will run for a billion years. We are limited only by the lifetime of the universe, whatever that might turn out to be.

The time requirement of the problem—that the device operate perpetually—is clearly unrealistic. As I just hinted, there is rather strong evidence that the universe will one day cease to exist altogether, taking our wonderful machine with it. But in a purely theoretical sense, the machine is potentially capable. It would, if it could, spin forever.

Now, if it were possible to eliminate all friction from an ordinary wheel, it, too, would spin forever, thanks to a law first discovered by the great British natural scientist Isaac Newton. This motion, while perpetual within any practical meaning of the word, is not the sort of motion we have just been discussing, as it produces no new energy. I will therefore call it type one perpetual motion. Earth satellites are essentially type one perpetual motion machines.

In type two perpetual motion we expect the device not only to exhibit potentially eternal motion but also to produce energy while doing so. In my opinion, type two perpetual motion is the more exciting of the two kinds.

Can the wheel in my mind's eye do actual work? Think of my wonderful wheel again, now writ large.

It will be found inside a secret government building somewhere in the desert. The wheel will be forty stories tall and made of more than a million tons of steel, Kevlar, diamond, and other awesome materials. We'll take an elevator to the fifteenth floor of the building, near the hub. We'll stare in stupefaction from our observation window as, one by one, the great spokes swing by, each with a great steel ball rolling along it with a terrifying rumbling noise. The secret installation sounds like a cosmic bowling alley.

Now, just before the great wheel reaches that critical speed where centrifugal force begins to eat away noticeably at its acceleration, a giant dynamo engages gears with the wheel, and outside the building power lines surge with millions of kilowatts of power, all of it apparently free.

That's what I mean by energy. Even a type two machine that produces the tiniest excess of energy may nevertheless be scaled up to almost any size—or multiplied a thousandfold by mass production. Indeed, any macroscopic machine that runs "forever" under ordinary circumstances must be overcoming frictional forces and must be type two.

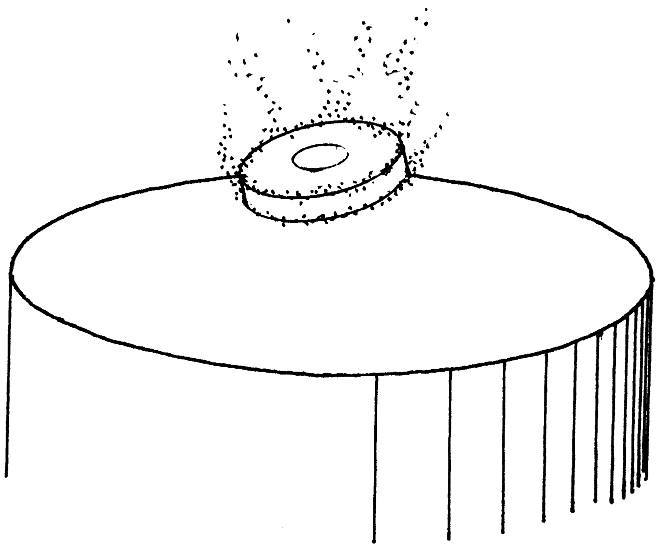
When scientists say that perpetual motion is impossible, they mean only that a machine that produces more energy than it consumes (typically none) is impossible. Type two machines are impossible. The reason for the impossibility lies with a fundamental tenet of modern physics, the law of conservation of energy. I will come back to this law later, subjecting it to a scrutiny it rarely receives. (Perhaps there is a flaw somewhere.)

In the meantime, we have been examining "machines" in the ordinary sense of the word, macroscopic systems of metal, plastic, even wood. But the microscopic world is inhabited by other systems: atoms in a state of apparent perpetual motion, electrons that whirl incessantly about nuclei, albeit in an unmanifest state, even at absolute zero. They are not producing any new energy, but their motion appears to be truly perpetual. After all, there is nothing to "wear out."

From this point of view, the closest we have yet come to a type one perpetual motion machine, here on Earth, is a doughnut-shaped ring of some hi-tech superconducting material in which electrons have been set circulating. Back in the 1980s, newspapers and science magazines frequently showed pictures of a superconducting ring floating above a magnet. How did it work?

The doughnut or torus is made from one of the latest metallo-ceramic hybrid materials that "superconducts" electrons at a temperature that is low, but not too low to be achievable in a laboratory. Superconduction works like ordinary conduction except that electrons move through the material without the production of heat (and subsequent loss of energy) that accompanies normal conduction.

Electrons in such a ring are set in motion by placing the ring in a strong magnetic field. The apparent levitation of the ring is not part of



Superconducting ring

the perpetual motion of the electrons inside it. Perfectly balanced, the ring does not move nor even rotate. The levitation is merely a macroscopic signal that the electrons inside it are circulating, something that newspapers never explained and science magazines rarely.

The Lorentz effect is a well-known physical phenomenon in which an electron, moving at right angles to a magnetic field, experiences a sideways force in a direction that is simultaneously perpendicular both to its direction of travel and to the field. Continually bending in the same direction, it travels in a circle and will do so as long as the magnetic field is present. At the same time, electrons traveling in a circle generate a magnetic field of their own. In the case of the ring, merely moving it into the magnetic field sets the electrons in motion. The proof that they do not slow down, as they would if the ring were made of an ordinary conductor, is the continuing opposing force that the circulating electrons set up. The magnetic "north" of the magnet is opposed by a balancing "north" generated by the electrons in the superconducting torus. It floats eerily in the magnet's field.

Left to themselves under ideal conditions, the electrons might be coaxed into circulating for arbitrarily long periods of time. This is perpetual motion of sorts, but it involves a microscopic motion within a macroscopic object that does not actually move, but remains suspended. In any event, the electrons have no frictional or other opposing forces to overcome. If they did, they would quickly grind to a halt, and the magic ring would fall with a rattle to the magnet beneath it. It follows that the circulating electrons are incapable of producing any new energy, and the superconducting torus exhibits, at best, type one perpetual motion. In the traditional search for perpetual motion, only machines with macroscopic motion need apply.

It fascinates many people, particularly mathematicians, how various physical theories interact in ways that their framers might not have foreseen. When they do, the result is rarely a contradiction. For example, suppose some bright young scientist developed a gravity-blocking material.

Such a device was used by the famous British science fiction writer H. G. Wells in his novel *The First Men in the Moon*. Two adventurous souls sit inside a special sphere equipped with food, air, and the other necessities of life. Two sets of blinds made of "cavorite" (after one of the adventurers, a scientist) are used to propel the sphere toward the Moon. By simply drawing the Earthside blind, gravity is blocked and the sphere floats upward, attracted by the Moon's gravity. Close to the Moon, the hardy explorers simply close the Moonside blind and open the Earthside one to slow the sphere to a gentle landing.

Is a material like cavorite possible? If it were, I could instantly invent a perpetual motion machine. I would simply turn a bicycle upside down and place the cavorite sheet on the ground below the front half of the front wheel. The portion of the wheel screened from gravity by the cavorite sheet would be weightless, and the heavier half would begin to turn Earthward. The wheel would therefore spin, faster and faster, until it reached its mechanical limits. Moreover, it would produce energy and be a true type two perpetual motion machine. The resulting contraption appears to contradict the laws of conservation of energy—until we realize that it resulted from the assumption that cavorite is possible. Cavorite or any material or device with the same property must therefore also be impossible. We have thus discovered a strange link between perpetual motion and gravity!

But is perpetual motion really impossible? If so, does this not represent an intolerable limitation on our freedom?

TYROS AND TRICKSTERS

The wheel on page 12 was actually dreamed up in the 1640s by Edward Somerset, the second marquis of Worcester. He had a device built, a monstrous wheel with forty 50-pound balls (weighing in total 1 ton) that rolled along curved spokes of wood. In 1648 he demonstrated his machine to King Charles I and members of the royal court in the Tower of London. Somerset was wealthy, but not exactly a dilettante. He contributed to the early development of the steam engine by designing a two-chambered engine that could pump water using the action of steam to create a partial vacuum.

He describes the occasion of the royal inspection of his wondrous machine as follows: "The wheel was fourteen foot over, and forty weights of fifty pounds apiece. Sir William Balfour, then Lieutenant of the Tower, can justify it with several others. They all saw that, no sooner these great weights passed the diameter-line of the lower side, but they hung a foot further from the centre, nor no sooner passed the diameter-line of the upper side, but they hung a foot nearer. Be pleased to judge the consequence."

Whatever the "consequence" was, it seems doubtful that the king or any of his courtiers saw perpetual motion that day. A wheel of such size and weight, once set in motion, might turn for a considerable time before stopping. And if, long after the royal party had departed, the wheel slowly came to a halt, the marquis might attribute the phenomenon to poor construction. All evidence points to Somerset's sincere belief that he had, indeed, achieved perpetual motion.

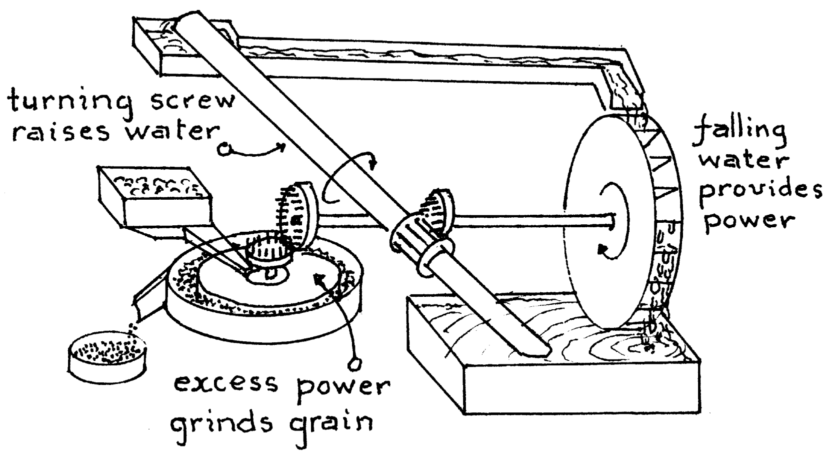
Was there a link between the development of the steam engine and the craze over perpetual motion? Did people once think that if only they were clever enough, they could dispense with the messy boiler and the need for fire? Probably not. The dream of perpetual motion goes back much earlier than the seventeenth century.

An intriguing passage in a seventeen-hundred-year-old Sanskrit book on astronomy, the *Siddhanta Ciromani,* describes a self-turning wheel. Its outer rim or tread was to be drilled with equally spaced holes that (probably) pointed not toward the axis, but a little ahead of it, so to speak. Each hole was half filled with mercury and then sealed. The passage claims that if the wheel was properly supported, it would turn forever.

In spite of the similarity in principle behind the Indian device and the wheel I have been describing, it is doubtful that any of the European Renaissance inventors were familiar with the Sanskrit book. Mark Antony Zimara, an Italian physician and astrologer, published a treatise on diseases of the body in about 1520. As an incidental enclosure, he included a description of a perpetual motion machine (something that few medical texts could get away with these days). The machine consisted of a large fan, which, when it turned, worked several levers that, in turn, operated three giant bellows. The bellows, in case you hadn't guessed by now, blew directly on the fan to keep it going. No record exists of anyone attempting to build Zimara's machine, and Zimara himself may have had lingering doubts, as this translation from the Latin hints: "This, perchance, is not absurd, but is the starting point for investigating and discovering that sublime thing, perpetual motion, a starting point which I have not read of anywhere, neither do I know of anyone who has worked it out."

Most of the early designs for perpetual motion machines in the Renaissance involved feedback of one kind or another: the motion of component A serves to keep component B moving, which, in turn, powers A.

In 1618, this principle was actively employed by Robert Fludd, an English physician and philosopher. He designed a closed-cycle grinding mill for which the motive power came from an endlessly circulating flow of water over a mill wheel. The wheel not only ground the grain but also powered an Archimedean screw, as in the illustration.



Fludd's closed-cycle grinding mill

Once set rotating, the screw brought the water from a discharge basin up to a reservoir, where it could begin its trip to the wheel all over again. Clearly, Fludd thought that somewhere in the cycle of the mill's action, the system would gain energy. Would the energy of the falling water be greater than the energy it took to transport the water to the top of the millrace? It is generally easier to propose a grandiose scheme than to carry it off. Fludd made no attempt to build his perpetual mill. If it had worked, Fludd's machine would have been a good example of a type two perpetual motion machine.

Many clergy of seventeenth-century England were also amateur scientists and inventors. Bishop John Wilkins of Chester designed a scheme very similar to Fludd's but went to the trouble of actually building it. He had the integrity to make his failure public: "When I first thought of this invention, I could scarce forbear, with Archimedes, to cry out 'Eureka! Eureka!' it seeming so infallible a way for the effecting of the perpetual motion that nothing could be so much as probably objected against it; but, upon trial and experience, I find it altogether insufficient for any such purpose, and that for two reasons: (1) The water that ascends will not make any considerable stream in the fall. (2) The stream, though multiplied, will not be of force enough to turn about the screw."

One of the most famous perpetual motion machines was constructed by a Polish German engineer, Johann Ernst Elias Bessler, who styled himself as "Orffyreus." In the 1710s, all Europe was abuzz with news of Orffyreus' amazing perpetual wheel. It was housed in a room of a castle that belonged to the landgrave of Hesse-Cassel at Wissenstein. The landgrave allowed Orffyreus to set the wheel in motion, then locked the room in which it was kept, with the landgrave's own official seal on the lock. At the end of two months, the story went, the room was opened, and there was Orffyreus' wheel, still turning at a good clip.

Many distinguished philosophers, engineers, and scientists came to view the amazing machine, and all went away shaking their heads in wonder. Writing to the French philosopher Jean Desaguliers, Baron Fisher described what he saw: "The wheel turns with astonishing rapidity . . . twenty turns a minute. This I noted several times by my watch, and I always found the same regularity. An attempt to stop it suddenly would raise a man from the ground. Having stopped it in this manner it remained stationary (and here is the greatest proof of perpetual motion). I commenced the movement very gently to see if it would of itself regain its former rapidity, which I doubted; but to my great astonishment I observed that the rapidity of the wheel augmented little by little until it made two turns, and then it regained its former speed."

Orffyreus' wheel was 12 feet in diameter, 14 inches wide, and had a rather large axle, about 8 inches thick. The entire wheel was covered with a waxed cloth to conceal the inner workings from those who might seek to steal the great Orffyreus' idea. After receiving a handsome gift from the landgrave, Orffyreus showed his patron the interior of the machine. He had extracted a solemn oath from the landgrave that he would never divulge the marvelous mechanism.

Perhaps what he saw roused the landgrave's suspicions. And perhaps enough skeptics of the wheel's operation remained to prompt him to launch an investigation. He hired the Dutch philosopher and engineer Professor s'Gravesande of Leyden to investigate. We have the professor's findings in a letter to Sir Isaac Newton. He found the wheel "covered over with canvas, to prevent the inside from being seen. Through the centre of this wheel runs an axis of about six inches diameter, terminated at both ends by iron axes of about three-quarters of an inch diameter upon which the machine turns. I have examined these axes, and am firmly persuaded that nothing from without the wheel in the least contributes to its motion. When I turned it but gently, it always stood still as soon as I took away my hand; but when I gave it any tolerable degree of velocity, I was always obliged to stop it again by force; for when I let it go, it acquired in two or three turns its greatest velocity, after which it revolved for twenty-five or twenty-six times in a minute."

Orffyreus, who was apparently not informed of the inspection, flew into a rage at the news, went to the castle, and smashed his marvelous engine beyond all repair. He never built another wheel, as far as I know, and no one ever explained how the wheel was able to turn so long without an outside source of energy. Suffice it to say that Orffyreus was trained as a clockmaker. It might be suspected that the extraordinarily thick axle held the secret. In any event, the need to conceal a secret source of energy would be covered by the same story as the need to protect a genuine idea. After all, a working mechanism would be worth, ultimately, all the wealth of the world, and no attempt to conceal its secret, however elaborate, could be considered unreasonable.

In the nineteenth century the search for perpetual motion intensified, amounting almost to a craze. At the same time, the skeptical eye had been honed to a point that would have made an Orffyreus impossible. The scene shifts to America, where, in 1812, a man named Charles Redheffer appeared in Philadelphia with a curious machine that, he claimed, would never stop. The public, eager for wonders, flocked to see the machine. Bets, some of them quite large, were made over the proof or disproof that Redheffer's machine actually worked as claimed.

The intrepid inventor applied to the Pennsylvania legislature for what today we would call a research grant. To inspect Redheffer's machine to determine the feasibility of perpetual motion, the legislature duly dispatched eight commissioners to examine the device. When they arrived at the building that housed the machine, they found that the room containing the machine was locked and Redheffer was nowhere in sight.

Through a barred window they saw a turntable mounted on a spindle shaft and apparently powered by two miniature trucks that rested, motionless, on two inclined planes, respectively. Each truck contained two small weights that, it was said, provided the actual power owing to their attraction by gravity. Although the trucks did not move, they were attached by levers to the spindle. The levers thus transmitted the force of gravity to the spindle, turning it—or so Redheffer claimed.

It is amazing that so many people were taken in by this machine. Since the little cars did not move relative to the large supporting wheel, nor the wheeled inclined planes, it would be utterly mysterious to a layperson untutored in mechanics how the thing worked. (It would, of course, be even more mysterious to a knowledgeable engineer.) Perhaps the gimcrack nature of the machine, coupled with the hypnotic effect of the slowly turning wheel, succeeded in convincing not only the lay visitor but the professional engineer as well. I mean, there it was, *turning*.

The illusion was reinforced by Redheffer or whomever he hired to operate the machine. Removing the little weights brought the device to an immediate halt.

The inspection by the commission of engineers employed by the legislature included a gentleman named Nathan Sellers, who brought his son along to see the machine. In the course of the inspection through the barred window, the son tugged on his father's coat. "See, Papa, the gearing looks wrong." The lad was referring to the birdcage gear that was supposed to be driven by the great wheel. The boy pointed out to his professional father, as well as to the other gentlemen present, that the wear on the wooden teeth of the birdcage gear was on the wrong side.

In other words, the gear was driving the great wheel, not the other way around.

Saying nothing to Redheffer, Sellers consulted later with Isaiah Lukens, an engineer and skilled mechanician with the Franklin Institute in Philadelphia. Sellers described the machine well enough for Lukens to make an almost exact copy. This would function as a lure to draw Redheffer out and so expose the fraud.

Although Lukens could not know what ultimate source of power Redheffer drew upon to keep his machine in motion, he devised one that was no doubt equally subtle. Under the handsome wood paneling below the great wheel, he installed a small spring motor that could be wound up by turning one of the wooden knobs on top of the ornamental framework that surrounded the machine. Thus a custodian could approach the exhibit as the machine was running down, draw out a rag and pretend to dust the machine, in reality turning the secret knob a few times as he pretended to polish it.

A demonstration of the duplicate machine was arranged by Sellers and Lukens, and they invited Redheffer to attend. Lukens had even arranged that the duplicate machine would also halt when the weights were removed. The clockwork motor concealed in the machine did not drive the birdcage gear, but the main axle itself, transmitting its force through friction that was barely sufficient with the weights in their cars. But with the weights removed, the friction dropped below the critical value, and the turning of the motor had no effect on the big wheel.

Redheffer could not conceal his amazement at the device shown to him by these sober, respectable citizens. Privately he offered Sellers a great deal of money if only he would reveal the principle by which the machine operated. Sellers may have replied, “Why, the principle is the same as that employed by your own good self: chicanery!”

Exposed, Redheffer decided to move his exhibit to New York. He had fleeced thousands of Philadelphians out of their one-dollar admissions (a good deal of money in those days) and could depend on the poor communications of the day to delay news of the scam.

Unfortunately for Redheffer, New York was not only the home of a much larger flock of potential believers but also the lair of one of America’s foremost engineers, Robert Fulton. Although he refused at first to attend the exhibit of what he considered to be an obvious fraud, friends prevailed upon him, and Fulton finally gave in. As soon as he entered the room to inspect the machine, he cried, "Why, this is a crank motion!" He meant that the unevenness of the turning sound from the device indicated that someone, somewhere, was turning a crank.

He denounced Redheffer, who happened to be present, on the spot. Redheffer blustered and became angry. Fulton said he would repay Redheffer in full for all damages and proceeded to attack two supports that were presumably meant to steady the machine against one wall. Inside one of the supports, Fulton discovered a catgut drive belt. Like bloodhounds on the scent, Fulton and his cronies sniffed at a hole in the wall through which the drive belt passed. They forced their way through a series of rooms until they came, according to one account, upon "an old man with a long beard who displayed all the signs of having been imprisoned in the room for a long, long time. The man had no notion of what was happening and sat there on a stool gnawing on a crust with one hand and turning a crank with the other."

Although surely a romantic exaggeration, the discovery of an accomplice put the crowd into a rage. They destroyed Redheffer's marvelous machine, and the great inventor had to flee for his life.

Perpetual motion machines continued to be the stock in trade of tricksters and con artists throughout the nineteenth century. In the 1850s, an engineer from Connecticut, C. P. Willis, made an elegant horizontal toothed wheel of brass that was enclosed, with the rest of the mechanism, in a glass case, where the curious could ponder the endless motion of the wheel. As the brass wheel spun, it engaged a gear that turned a flywheel that communicated with the first wheel by a system of pulleys. It was the old dream of A drives B while B drives A.

Willis charged people to inspect his machine, first in New Haven and later in New York. An alert patent attorney, however, noticed a strut near the flywheel that had no apparent function. He discovered that Willis had arranged a supply of compressed air, fed into the strut, to blow on the flywheel and keep it in motion.

It would be fascinating to view a vast museum exhibit of all the perpetual motion machines ever built or even imagined. There would be hundreds, if not thousands. There would be not only overbalancing wheels and feedback water mills but also sponge machines employing capillary action, machines driven by magnets, and numerous other devices. They could all be started at once, and the great exhibit hall would be filled with a kind of mechanical sigh of despair as collectively and one by one the machines ran down or their fraudulent energy sources were cut off.

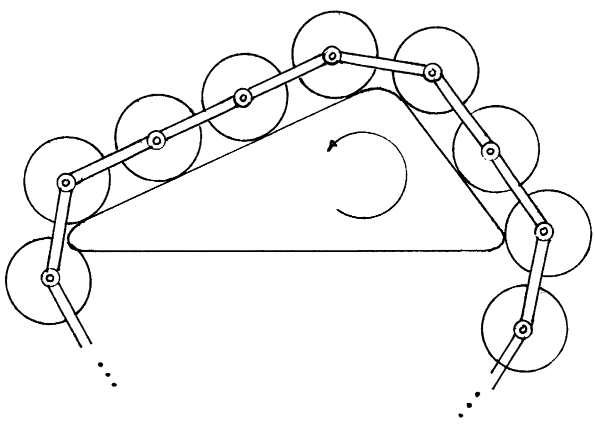
PROOFS OF IMPOSSIBILITY

Newton's third law states that a body at rest or in a state of uniform motion will continue in the state unless influenced by some external force. In other words, all motion is perpetual in the absence of external forces. Paradoxically, type one perpetual motion is built into the very foundations of classical physics.

As for type two, or true perpetual motion, one may analyze each device in turn and prove that it could not possibly be producing more energy than it consumes. Such proofs, because of the complexity of some devices, may involve considerable labor. In other cases, as below, the analysis is relatively straightforward. I will present an analysis of a device first described by the Dutch philosopher/mathematician Simon Stevinus at about the turn of the sixteenth century.

As far as I can tell from available sources, Stevinus may have "invented" the machine that appears in the figure to demonstrate the impossibility of perpetual motion (in this one case), or he may have come upon the design in sources that are no longer available. I prefer to think the latter.

The machine shown here consists of fourteen "rollers," essentially cylinders that are linked by a flexible chain connecting their axles. Four



The two-track roller machine

rollers occupy the long ramp on the left of the figure, while two rollers rest upon the shorter ramp on the right. It is easy to imagine that the four rollers on the left exert a greater force on the endless chain than do the ones on the right. After all, there are four of them. As Stevinus observed, the eight rollers (not shown in the figure) that hang under the ramps make no contribution to the resulting perpetual motion because they exert an equal force on both the right and left portions of the chain. My analysis will be a simplified and more modern version of the one given by Stevinus.

Let us call the long ramp A, the short one B. We do not know the exact lengths of these ramps, but if we designate them by the symbols a and b, respectively, then $a = 2b$, since the long ramp accommodates twice as many rollers as the short one.

To analyze this particular machine, we will search for the motive power that would result from an imbalance of forces among the rollers. Such an imbalance would surely manifest at the portion of chain joining the four rollers on the long ramp to the two rollers on the short one. At that point, the force due to gravity manifested along the long ramp will presumably be greater than the force along the short one.

We use the trigonometric cosine function to express the action of gravity along the angle made by either ramp with the vertical. In other words, although the force of gravity acts downward on a roller, the ramp

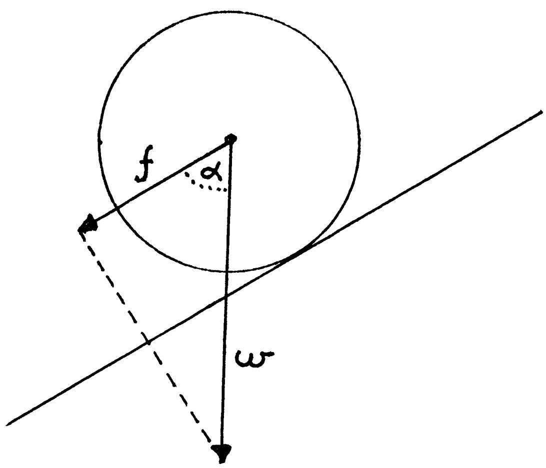


Diagram of forces on a roller

prevents it from moving in the downward direction. The force must act along the ramp, that being the only direction in which a roller can move.

In this right-angled triangle the vertical arrow represents the downward force on a roller due to gravity. If each roller weighs *w* grams (or pounds), the downward force is *w.* The sloping arrow, on the other hand, represents the component of gravity acting on the roller in the downramp direction. We'll call this force *f.* If α is the angle made by the ramp with the vertical, then the cosine of α, written $\cos(\alpha)$, is simply the ratio of the sides f/w. In other words, $\cos(\alpha) = f/w$, so that *f,* the downramp force, equals $w\cos(\alpha)$.

If α is the angle made by the long ramp with the vertical, we'll suppose the short ramp makes the angle β. The downramp force to the left must result from the weight of four rollers, as resolved by the cosine operator:

$$4w\cos(\alpha).$$

Similarly, the downramp force in the other direction, with just two rollers, must be:

$$2w\cos(\beta).$$

We can now eliminate the cosine function by recognizing that it represents the ratio of two distances. In this case, $\cos(\alpha) = c/a,$ while $\cos(\beta) = c/b.$ The two forces may now be written as follows:

$$\begin{aligned}\text{force along shallow ramp} &= 4w\cos(\alpha)\\ &= 4wc/a,\end{aligned}$$

while

$$\begin{aligned}\text{force along steep ramp} &= 2w\cos(\beta)\\ &= 2wc/b.\end{aligned}$$

As we observed earlier, the distances *a* and *b* have the relationship

$$a = 2b.$$

This leaves us with the

$$\begin{aligned}\text{force along shallow ramp} &= 4wc/a\\ &= 4wc/2b\\ &= 2wc/b,\end{aligned}$$

which just happens to be the force along the steeper ramp. Obviously, the rollers will not begin, of themselves, to move. Nor will they develop any additional energy as a result of moving. If friction is reduced to a minimum, the rollers may continue to circulate up one ramp and down the other for a while, but eventually they will slow down and stop.

A similar analysis could be made of the Somerset machine, except that the geometry leads to more complicated expressions. Suffice it to say that for each position in which the wheel may be overbalanced in one direction, there is another position in which it overbalances by the same amount in the opposite direction.

If a mere glance at the first position was enough to convince Edward Somerset, as well as thousands of people after him, that the wheel would turn forever, a glance at the second position might have given him pause. If popular illustrations of the wheel had included both positions, the machine might not have inspired such perpetual emotion.

PHYSICS HAS ITS SAY

There are general physical laws that imply that type two perpetual motion is impossible. The laws may even be cast in mathematical form, as we will presently see. Thus, in analyzing the various proposals for perpetual motion machines, we may always say, "There's no need to analyze these things in detail. There are physical laws that say such machines can never work."

On the other hand, we may also analyze each machine mathematically, employing the simplest physical concepts such as leverage and force, concepts that have nothing to do with the grand laws we are about to explore. Applying these concepts and incorporating them into a mathematical analysis of each device always lead to the same conclusion: it cannot work as advertised.

This is a very strange phenomenon when you think about it. It implies a consistency between our applied mathematical analyses and a general fiat about conservation of energy, a consistency that cannot be explained within any framework of knowledge currently available to us. If we had no notion whatever that all physical systems were constrained by the law of conservation of energy, we would still be reaching conclusions of impossibility for these machines based purely on applied mathematical reasoning.

CONSERVATION OF ENERGY

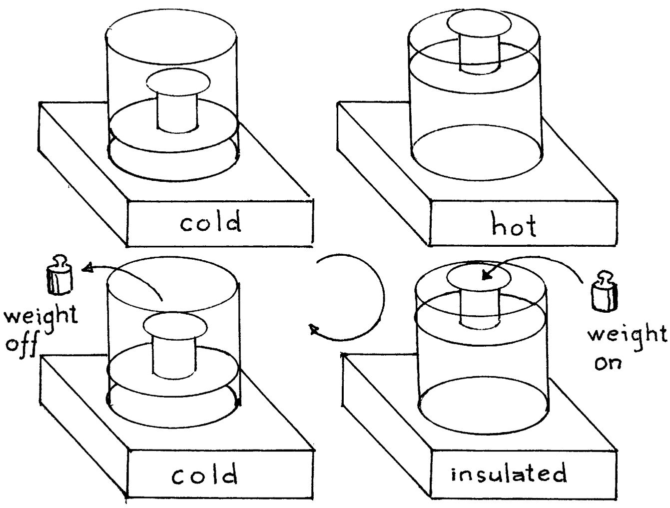
The historical beginnings of the law of conservation of energy reveal an ironic twist. The law was inspired in part by the belief of early scientists, beginning with Galileo Galilei of Italy, that perpetual motion was impossible. Based as it was on the continuing failure of such devices to work, we cannot say that the belief was exactly arbitrary. However, it led to some astonishing progress from the seventeenth to the nineteenth centuries.

Galileo, for example, analyzed the motion of bodies rolling down inclined planes. He concluded that a ball rolling down a straight ramp from top to bottom must reach the same velocity (friction aside) at the point where the ramp met the tabletop as it would if dropped directly from the release point onto the table. If this were not so, he reasoned, type two perpetual motion would be possible. Suppose the ball came off the ramp at a greater speed than it would develop by merely falling vertically. In such a case it could be deflected upward by a suitably arranged elastic barrier at the bottom of the ramp. The ball would bounce upward, rising higher than the point from which it started. If it landed on a second ramp, it could roll back to its starting point on the first ramp with even greater energy than on the first occasion.

On the other hand, if the ball came off the ramp with a slower speed than it would have if dropped (friction aside), the procedure outlined above could be reversed. If the ball were dropped from the same height as the starting point on the ramp, directly onto the same barrier, now arranged to deflect the ball directly up the ramp, it would travel beyond the release point. In this case it could be arranged for the ball to fall through a second hole, striking the tabletop with even greater force than it did on the first occasion and so on, ad infinitum.

An able experimenter, Galileo tested his hypothesis and found it to be correct.

Two hundred years later, Sadi Carnot, a brilliant young French scientist, devised a conceptual scheme that gave direct insights into the relationship between mechanical work and another form of energy: heat. In 1824 he imagined a peculiar apparatus consisting of a gas-filled cylinder with a piston that fitted the cylinder so perfectly that no gas could escape. Two heat reservoirs, one at a high temperature, the other at a low one, served to add heat energy to the cylinder or to remove it when the cylinder was placed over them.



Carnot's "gedanken" experiment

Placed over the cold reservoir, the piston would take up a particular position within the cylinder. Placed over the hot reservoir, the cylinder would gain heat and the temperature of the gas would rise, accompanied by a rise in pressure. The piston would therefore rise in the cylinder to a new, higher position.

After heating his conceptual cylinder, Carnot moved it to an insulating pad, where the piston's position remained unchanged. Carnot then placed a weight on the piston, causing it to descend into the cylinder back to its original position. Next he moved the cylinder to the cold reservoir, simultaneously removing the weight. The position of the piston did not change in the end. The tendency of the piston to rise, thanks to the removal of the weight, was exactly counterbalanced by a drop in pressure within the cylinder, thanks to the removal of heat.

Heat had therefore been transferred from the hot reservoir to the cold one, the exact amount being equivalent to the work done by gravity acting on the weight and causing the piston to compress the gas. Carnot concluded that there was a direct and simple relationship between changes in heat energy in a cylinder and the amount of work done on it. Today we call this conceptual process the Carnot cycle, fundamental for understanding everything from automobile engines to refrigerators. Heat and mechanical energy are simply two forms of energy, and one could be converted into the other. Except for loss through leakage of heat or friction acting on mechanical motion, energy was conserved. The equations governing the relationship of mechanical and heat energy remain fundamental to thermodynamics today.

It is germane to our central theme that the experiments of Galileo and Carnot had a purely logical (i.e., mathematical) structure. Whether the experimental system involved rolling spheres or heated gases, a logical deduction could be made from the assumption that energy was conserved. Here is how the system ought to behave. Voilà! It does. Interestingly enough, the conclusion that conservation of energy is real is not actually a logical deduction from the experimental results. Only if the experiment had turned out negatively could either Galileo or Carnot have said, "Alas, energy does not appear to be conserved!" Instead, we can only say that the experimental outcomes "support" the idea that energy is conserved. That is the real difference between inductive and deductive science.

In the next chapter we will see an experiment in which no actual test is necessary, the famed "gedanken" experiments of Einstein and associates.

A mere twenty-three years after Carnot published his analysis of heat engines, the German physicist Hermann von Helmholtz presented the first formal statement of the law of conservation of energy. He began his address to the Physical Society of Berlin by declaring that perpetual motion was axiomatically impossible. The general law he proposed stated that energy could neither be created nor destroyed, but only changed from one form to another.

These early foundations of the law of conservation of energy might therefore be cited as a form of circular reasoning: perpetual motion is impossible because conservation of energy is true; conservation of energy is true because perpetual motion is impossible. On the other hand, if perpetual motion really was impossible (and empirical experience strongly suggested this to be the case), then additional weight was added to the newly minted law.

One could say that the search for perpetual motion, by focusing the attention of scientists on the intimate energy transactions within various machines, caused them to consider the same transactions within the wider scope of all physical systems. If the search for perpetual motion was not exactly the mother of conservation of energy, it was certainly the midwife. In any event, we have come to the final act of this drama: What, exactly, does conservation of energy mean?

When we talk about the conservation of energy, we must understand what we mean by "energy." Everyone understands that a fast-moving object such as a speeding automobile has a lot of energy. Just try to stop one. All motion involves the form of energy we call "kinetic." Even very small objects such as atoms carry kinetic energy when they move.

Another form of energy is heat. In most physical bodies it is stored in the form of kinetic energy, namely the motions of the many atoms that compose them. Heat may also take the form of radiation, being received or emitted by a body in the infrared portion of the electromagnetic spectrum. Machines with moving parts constantly generate heat from friction. This heat is normally conducted or radiated away.

There are other forms of energy as well. Potential energy is the energy of position. It depends on an external force field such as gravity, electricity, magnetism, or what have you, to be expressed. A car on a 100-foot cliff may not be moving, but relative to the ground below it, the car carries the potential to develop a lot of kinetic energy. Push it off the cliff and you will witness the effect of the Earth's gravitational field on the vehicle. Its potential energy will be converted into kinetic energy that is fully equivalent to the energy of the car when it speeds down the highway. All bodies, from planets to atoms, have potential energy of one kind or another.

Energy, as it turns out, inheres not only in the kinetic or potential behavior of physical bodies but in their very substance. Mass, according to Albert Einstein's celebrated formula, is equivalent to energy:

$$E = mc^2.$$

The "rest energy" of a physical body equals its mass multiplied by the speed of light squared. This speed, expressed in meters per second, is very large even when not squared:

$$c = 299{,}792{,}458 \text{ meters per second.}$$

This may be written approximately as 3×10^8 meters per second. A kilogram of mass of any kind (steel, pigeon feathers, or garden soil) would therefore contain 3×10^{16} joules of energy. A joule of energy is well within our ability to experience directly. For example, it takes about 10 joules of energy to raise a 1-kilogram (about 2.2-pound) rock 1 meter (a bit over 3 feet) above the floor. If the same rock were converted entirely into energy, it would produce approximately 3×10^{16} joules. If this energy were released all at once in the form of an explosion, it would rival a very large atomic bomb in its effect. (Although atomic bombs use much larger masses, only about 0.1 percent of their masses are converted directly into energy.) A more striking example would involve an ordinary penny, which, if it could be converted entirely into energy, would supply the power needs of the average house for a lifetime.

We therefore have three kinds of energy. The law of conservation of energy states that in every "isolated" physical system, energy may be transformed from one kind to another, but never created nor destroyed. A proposed type two perpetual motion device, if not allowed any energy input from the outside world, would qualify as an isolated system but would not be exempt from this law.

Put mathematically, the law of conservation of energy states that an isolated physical system must obey the following equation:

$$mc^2 + K + P + H = \mathrm{E} \text{ (a constant).}$$

Here, mc^2 is the rest energy of the system. Assuming that no nuclear processes are involved in a perpetual motion device, this quantity will remain constant, and I can move it to the right-hand side, absorbing it in a new constant, E'. The letters K, P, and H represent kinetic, potential, and heat energy, respectively, within the system. A simplified equation results:

$$K + P + H = \mathrm{E}' \text{ (another constant).}$$

In Somerset's wheel, we learned of two positions from which it would turn spontaneously, albeit in opposite directions. In either position, the potential energy was high and the kinetic energy was zero. As the wheel began to turn from the first position, its value of K would rise, exactly compensated by a drop in P. As it continued to turn, it would approach the second position, at which point P would again begin to rise and K to drop, again by an amount that compensated exactly for the drop in K.

Did I say “exactly”? If there were no friction at all, either from the air or from the axle, this would be true. The wheel would approach the second position, pause, then begin to turn in the other direction, replaying the previous scenario in reverse. In fact the wheel would develop perpetual motion, but not exactly what Somerset envisioned. It would enter a regime of perpetual (type one) motion.

Since the wheel must encounter friction as it turns, it would not quite reach the second position. Nor, in turning back, would it ever again reach the first. Instead, it would rock back and forth in gradually diminishing arcs until it finally came to rest in what physicists call a position of equilibrium. Full stop. The energy robbed from the wheel by friction would be converted into heat H, some of it retained in the wheel, the rest radiated away as electromagnetic energy. A diagram of the wheel in this final position would create very little excitement in potential inventors.

IS THERE A WAY AROUND IT?

The final question leaves us with an unsolved mystery, one that mathematics would seem unable to address because it involves a very deep structural property of the universe in which we find ourselves. How do we know that the law of conservation of energy is valid? As with other physical “laws,” we do not make deductions, but inferences.

The only loophole I can think of involves the exchange of energy and information. Maxwell’s demon, a conceptual imp named after the Scottish physicist James Clerk Maxwell, plays a prank on us by sitting astride a tiny doorway between two gas-filled vessels, A and B. The demon is small enough to observe directly the motions of individual gas molecules in either vessel. When he sees a molecule approach the doorway from vessel A, he opens the door and allows it to pass into vessel B. But when he sees a molecule approach the doorway from vessel B, he keeps the door closed. Thus, over time, pressure builds in vessel B until it contains virtually all the gas molecules.

A pipe that connects the two vessels passes through a turbine and, when a stopcock is opened, the turbine runs for a while until the pressures in the two vessels are once again equal. Again the demon sets to work, and again the turbine is run. Is this not—conceptually, at least—an example of type two perpetual motion?

The answer lies in quantum mechanics. As we will see in chapter 3, the kind of knowledge to which the demon is privy is impossible to obtain. He may know the position of a particle only to the extent that he is ignorant of its speed. He will not know when to open the door, in effect.

And even in quantum mechanics, energy is conserved, being quantized—that is, occurring in discrete packets called quanta. Each atom contains a certain amount of energy in the levels of excitation of its electrons. Should a single electron drop from a higher to a lower energy level, the atom loses energy. But the energy lost is emitted by the atom in the form of a quantum of energy that travels in the form of a wave until it is absorbed by some other atom. Quanta do not disappear, nor do they appear out of nowhere. Quantum mechanics is the most successful physical theory ever developed.

As one of the most general physical laws we have, conservation of energy pervades both the classical and the quantum mechanical branches of physics. In the classical domain, which for our purposes consists of Newtonian physics and relativity theory, no experiment ever performed has found a violation of this law. In fact, the mathematical accounting that goes into every physical calculation has never revealed a violation of either of the equations involving *K, P,* and *H*. Energy may be converted from one form to another, but it is never lost. Energy radiated away from one system will eventually arrive at another, to put it crudely. It is never lost in transit, nor does it ever appear out of nothing.

If someone, someday, designs and builds an actual type two perpetual motion machine, that person will not only become wealthy enough to make Bill Gates look poor, but also he—or she—would be entitled, I should think, to a Nobel Prize in physics.